6.4: Properties of Logarithmic Functions

***Product and Quotient Properties:** $m > 0, n > 0, b > 0, \text{and } b \neq 1$

Product: $\log_b(mn) = \log_b m + \log_b n$

Quotient: $\log_b \frac{m}{n} = \log_b m - \log_b n$

***Notice the bases are the same!**

***Power Property:** $m > 0,\ b > 0, \text{and } b \neq 1$, and any real number p.

$$\log_b m^p = p \log_b m$$

***Exponent becomes multiplier!**

Why… $\log_b(a^4) = \log_b(a \bullet a \bullet a \bullet a)$

$= \log_b a + \log_b a + \log_b a + \log_b a = 4 \log_b a$

***Inverse Property:** $b > 0 \text{ and } b \neq 1$

$$\log_b b^x = x \text{ and } b^{\log_b x} = x \text{ for } x > 0$$

***After the cancellations, the "x" remains!**

***One-to-One:** If $\log_b x = \log_b y$, then $x = y$

***Drop the logs if the bases are the same!**

Write each expression as a sum or difference of logarithms.

1) $\log_6 7x$

$\log_6 7 + \log_6 x$

2) $\log_2 \frac{17}{24}$

$\log_2 17 - \log_2 24$

3) $\log_5 \frac{3y}{2}$

$\log_5 3 + \log_5 y - \log_5 2$

Write as a single logarithm, then simplify.

4) $2\log_3 9 + \log_3 3$

$\log_3 9^2 \cdot 3$

$\log_3 243$

5

5) $\log_6 9 + \log_6 4$

$\log_6 36$

2

6) $4\log_5 x - 3\log_5 x$

$\log_5 x^4 - \log_5 x^3$

$\log_5 \frac{x^4}{x^3}$

$\log_5 x$

7) $\log_2 6 + \log_2 12 - \log_2 9$

$\log_2 \frac{6 \cdot 12}{9}$

$\log_2 8 = 3$

8) $3\log_4 2 + 2\log_4 4 - \log_4 2$

$\log_4 \frac{2^3 \cdot 4^2}{2}$ $\log_4 \frac{8 \cdot 16}{2}$ $\log_4 64$

3

Evaluate

9) $\log_5 5^2 + \log_4 64$

$5^x = 5^2$

$\log_5 25 + \log_4 64$

$2 + 3$

5

10) $8^{\log_8 12} - \log_2 2^3$

$\log_8 x = \log_8 12$

$x = 12$

$12 - 3$

9

$2^x = 2^3$